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Method of Applying Seismic Shear Force Coefficient A_i for a Twin-Tower Building and the Axial Force in the Rigid Beam Connecting the Towers

by

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Abstract

In order to conduct the first phase of the design of earthquake resistant buildings under the Japanese building code, seismic shear force Q_i must be considered, which is related to seismic shear coefficient A_i which is a vertical distribution factor. Generally, A_i is evaluated by the simple equation shown in the Japanese building code. However, it is limited to only buildings of typical shape. This paper presents the usage of seismic shear coefficient A_i for a twin-tower building due to horizontal vibration. Furthermore, the SRSS equation used to calculate the precise seismic shear coefficient $A_{i(SRSS)}$ is shown. Then, the values of Q_i and $Q_{i(SRSS)}$ (Q_i *is* result of A_i by simple equation and $Q_{i(SRSS)}$ is the result of $A_{i(SRSS)}$) are compared to determine whether the simple equation is safe or not by checking whether Q_i is bigger than $Q_{i(SRSS)}$. The comparison showed that the design deformation angle of the first story's column is 1/350, that of the top story's column is 1/200, and that of stories between the first and top are linear interpolated value, which is the safety condition in order to use the simple equation. Next, we applied the same condition to a twin-tower building to study the usage of seismic shear coefficient A_i. We also studied the axial force in the rigid-beam connecting tower1 and tower2.

Keywords: Twin-tower building, Seismic shear coefficient

1. Introduction

As the world's population continues to increase and people move from rural area to urban area. Then, the need of working and living space in urban area are required.

Changing horizontal space to vertical working and living space as much as possible is a solution of increasing demand in urban area. Not only high-rise building but also include twin towers building. Twin towers building is not only solution of the increasing demand of working and living space in urban area but it also be a landmark of area. For example, Petronas twin towers building in Kuala Lumpur, Malaysia.

In order to do the first phase design of earthquake resistance building by Japanese code, this type of structure will become more complicate, due to different of dynamic properties of both of towers. Then, A_i simple equation cannot use to consider safety seismic shear coefficient Ai.

In this paper, we investigate on usage of seismic shear coefficient A_i in case of twin towers building due to horizontal vibration in term of A_i from A_i simple equation. SRSS equation is indicated to calculate the precise seismic shear coefficient $A_{i(SRSS)}$. First, it is necessary to find out deflection of column of general shape building which give a safety condition $(Q_i > Q_{i(SRSS)})$ in case of using A_i simple equation by comparison of Q_i and $Q_{i(SRSS)}$. After that, we apply the same safety condition to twin tower case and consider usage of seismic shear coefficientA_i.

2. General Shape Building

2.1 *First phase design of earthquake resistant building by Japanese code*

The first phase design for earthquakes is a process of evaluating lateral seismic force Q_i of i-th story of a building. It can be determined by the following equation.

$$Q_i = C_i W_i \qquad (1)$$

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where

$$C_i = ZR_tA_iC_o \qquad (2)$$

$$A_i = 1 + \left(\frac{1}{\sqrt{\alpha_i}} - \alpha_i\right)\frac{2T}{1+3T} \qquad (3)$$

where

Q_i= seismic shear force at i-th story
W_i= carry weight of i-th story
C_i= shear coefficient at i-th story
Z = seismic zone factor
R_t= vibration characteristics factor
A_i= vertical distribution factor
C_o= standard shear coefficient
α_i= non dimensional weight of each story
T= natural period of structure (T=0.03h: steel structure)
h= building height

2.2 *Structural model and assumption*

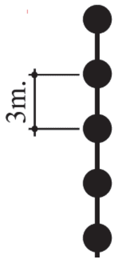


Fig.1 General shape building model

Figure 1 shows the lumped mass model of general shape building which has floor to floor height equals to 3m. Generally, floor load in each floor of building usually same. Then, in this research, all of masses are equal to 1 by ratio and connected by stiff column. In this research, the minimum height of building is 9m (3stories). Maximum height of building is 45m (15 stories), which is the building's height limit in order to do the first phase design by Japanese code. We assume that standard coefficient is equal to 0.2 ($C_0 \geq 0.2$ for allowable stress design against moderate earthquake) and Z is equal to 1 (Kanto area). Moreover, soil type is assumed as medium soil, then T_c is equal to 0.6 sec. It will give vibration characteristics factor shown in Fig.2.

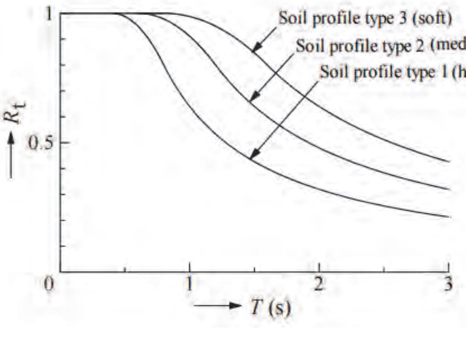


Fig.2 Vibration characteristics factor R_t

2.3 *Safety condition analyzing method*

In this study, values of Q_i and $Q_{i(SRSS)}$ are compared to ensure whether the simple equation is safety or not by checking Q_i is bigger than $Q_{i(SRSS)}$. Q_i can be determined by equation (1) and (2). Where A_i can be determined by $A_i$simple equation (3). In the same way, $Q_{i(SRSS)}$ can be determined by the following equation.

$$Q_{i(SRSS)} = C_{i(SRSS)}W_i \qquad (4)$$

$$C_{i(SRSS)} = ZR_tA_{i(SRSS)}C_o \qquad (5)$$

where the following Square Root of Sum of Squares (SRSS) equation (in reference (1)) is indicated to calculate the precise seismic shear coefficient$A_{i(SRSS)}$.

$$A_{i(SRSS)} = A'_{i(SRSS)}/A'_{1(SRSS)} \qquad (6)$$

$$A'_{i(SRSS)} = \sqrt{\textstyle\sum_{j=1}^{k}\left(\sum_{m=i}^{n} W_m\beta_jU_{mj}R_t(T_j)\right)^2}/\textstyle\sum_{m=i}^{n} W_m \qquad (7)$$

$$\beta_j = \textstyle\sum_{i=1}^{n} m_i\,U_{ij}/\sum_{i=1}^{n} m_i\,U_{ij}^2 \qquad (8)$$

where

n = number of story
W_i= carry weight of i-th story
β_j= participation factor of mode j
$R_t(T_j)$= vibration characteristics factor of natural period *T* of mode *j*
U_{mj}= mode vector of mode j
m_i= mass of i-th story

In this research, we consider vibration mode from 1stmode to 3rdmode; *k*=3

In order to determine vibration properties of the lumped mass system, the following generalized eigenvalue problem is calculated to get the natural circular frequency ω.

$$|-\omega^2\,[M] + [K]| = 0 \qquad (9)$$

Where $[M]$ is mass matrix which consists of mass by ratio of each story. In the process of structural design, seismic shear force Q_i must be less than allowable seismic shear force and the maximum deflection angle of column must not exceed 1/200 and assumption of design deformation angle θ_i give the stiffness matrix $[K]$, which consists of stiffness of column of each story can be determined by the following equation.

$$K_i = Q_i/\theta_i \qquad (10)$$

Moreover, Q_i shown in equation (10) can be determined by equation (1).

Because of mass matrix $[M]$ consists of mass by ratio. Then, generalized eigenvalue problem will give unrealistic natural circular frequency ω_{eigen} and also natural period T_{eigen}. In this research, natural period T_{eigen} of 1st mode of vibration from generalized eigenvalue problem is indicated to find out modification ratio (mr) by the following equation.

$$mr = T/T_{eigen}\,(1);\; T = 0.03h \qquad (11)$$

Modification ratio (mr) from equation (11) is used to modify natural period T_{eigen} of every j mode of vibration by the following equation

$$T_{modify}(j) = T_{eigen}(j) \times mr \qquad (12)$$

2.4 *Safety condition*

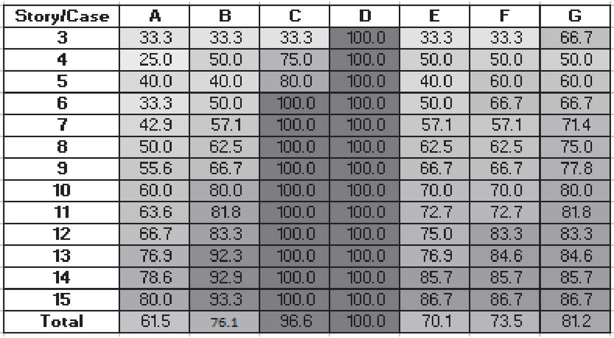
The safety condition analyzing method is processed by several cases of design deformation angle. Design deformation angle of the first story's column and the top story's column of each case are shown in the following table. Moreover, deflections of column at story located between the first and the top story are assumed interpolated value.

In this research, we consider safety condition indicated by safety percentage of all story from 3 story building to 15 story building. For example, in case of 3 story building, if $Q_1 > Q_{1(SRSS)}$,$Q_2 > Q_{2(SRSS)}$but $Q_3 < Q_{3(SRSS)}$, total safety percentage will equal to 66.7%. On the other hand, if all of $Q_\mathrm{i} > Q_{\mathrm{i}(SRSS)}$, total safety percentage will equal to 100%.

Table 1 Case of design deformation angle

Case	First story's column	Top story's column
A	1/200	1/200
B	1/250	1/200
C	1/300	1/200
D	1/350	1/200
E	1/200	1/250
F	1/200	1/300
G	1/200	1/350

Table 2 Comparison of total safety percentage of 3-15 story building in each case of design deformation angle (unit: percentage)



Story/Case	A	B	C	D	E	F	G
3	33.3	33.3	33.3	100.0	33.3	33.3	66.7
4	25.0	50.0	75.0	100.0	50.0	50.0	50.0
5	40.0	40.0	80.0	100.0	40.0	60.0	60.0
6	33.3	50.0	100.0	100.0	50.0	66.7	66.7
7	42.9	57.1	100.0	100.0	57.1	57.1	71.4
8	50.0	62.5	100.0	100.0	62.5	62.5	75.0
9	55.6	66.7	100.0	100.0	66.7	66.7	77.8
10	60.0	80.0	100.0	100.0	70.0	70.0	80.0
11	63.6	81.8	100.0	100.0	72.7	72.7	81.8
12	66.7	83.3	100.0	100.0	75.0	83.3	83.3
13	76.9	92.3	100.0	100.0	76.9	84.6	84.6
14	78.6	92.9	100.0	100.0	85.7	85.7	85.7
15	80.0	93.3	100.0	100.0	86.7	86.7	86.7
Total	61.5	76.1	96.6	100.0	70.1	73.5	81.2

Table 2 shows the safety percentage of 3-15 story building in each case of design deformation angle.

First, we analyze the safety condition by condition of case A (design deformation angle of the first story's column and the top story`s column angle is 1/200). Then, we found that it could not give safety condition. Next, we decrease the design deformation angle of the first story's column from 1/200 to 1/250, 1/300 and 1/350 as shown in case B, C, D. Then, we found that design deformation angle in case D can give safety condition. Next, we try to decrease the design deformation angle of the top story's column from 1/200 to 1/250, 1/300 and 1/350 as shown in case E, F, G. Then, we found that it could not give safety condition.

From the research, we found that case D (Design deformation angle of the first story's column equals to 1/350, that of the top story's column equals to 1/200 and those in between the first and top are linear interpolated value) is safety condition in order to use the simple equation (3).

3. Twin Towers Building

3.1 *Direction of movement*

Generally, movement direction of building due to the earthquake cannot be predicted such as the movement direction which are shown in Fig.3. It may move as case 1, case 2 or case 3. In this paper, we consider seismic shear coefficient A_i only in case 1 (both of towers move together in X direction).

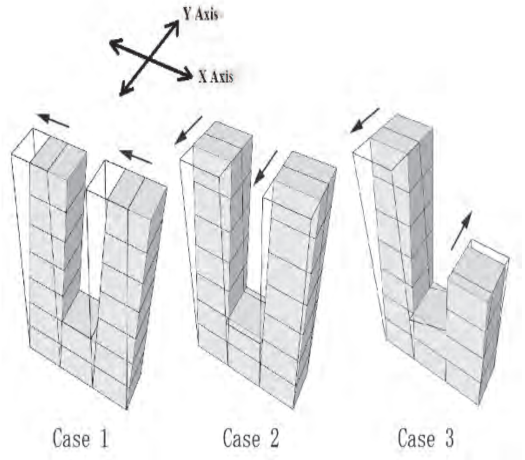


Fig.3 Case of movement direction of twin towers building

3.2 *Structural model and assumption*

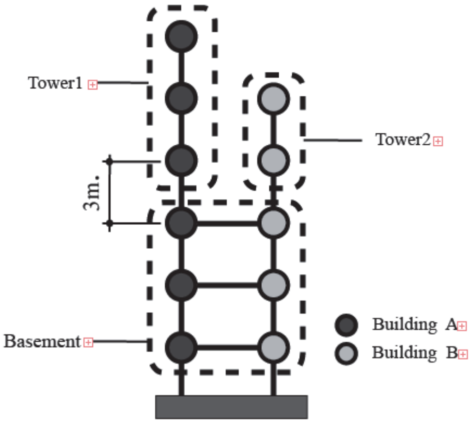


Fig.4a Twin towers building model

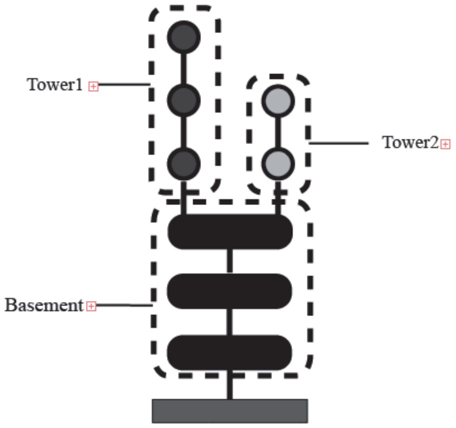


Fig.4b Twin towers analyze model

Figure 4a shows model of twin towers building. It consists of two buildings (building A and building B). In term of building zone, it consists of three zone; Basement, tower1 and tower2. Floor to floor height of building is equal to 3m. All of masses of tower are equal to 1 by ratio. All of masses of basement are equal to 2 by ratio. All of masses in each building are connected by columns. Furthermore, both of building are connected by rigid-beam and assumed that mass which is connected by rigid-beam will have a same movement under vibration. In this research, the minimum height of each building is 9m (3story). Maximum height of each building is 45m (15 story), which is the building's height limit in order to do the first phase design by Japanese code. Building A and B are connected by rigid-beam as least 3 stories. We assume that standard coefficient is equal to 0.2 ($C_0 \geq 0.2$ for allowable stress design against moderate earthquake) and Z is equal to 1 (Kanto area). Moreover, soil type is assumed as medium soil, then T_c is equal to 0.6 sec. It will give vibration characteristics factor which show in Fig.2

Figure.4b shows lumped mass model of twin tower model, which is used to analyze in process of safety condition analyzing method. In term of assumption, both of model are same. On the other hand, only mass in basement zone are equal 2 by ratio.

3.3 *Usage of seismic shear coefficient A_i analyzing method*

In order to consider usage of seismic shear coefficient in case of twin towers building, we apply condition of case D in general shape building model to twin tower model. Design deformation angle of the top story of taller tower is 1/200. Even height of tower1 and tower2 are different, design deformation angle at the same story of each tower are same. In the same way as safety condition analyzing method of general shape building, values of Q_i and $Q_{i(SRSS)}$ are compared to ensure whether the simple equation is safety or not by checking Q_i is bigger than $Q_{i(SRSS)}$.

Table 3 Consideration of carry weight of i-th story W_i (unit: weight by ratio) of lumped mass model in Fig.4

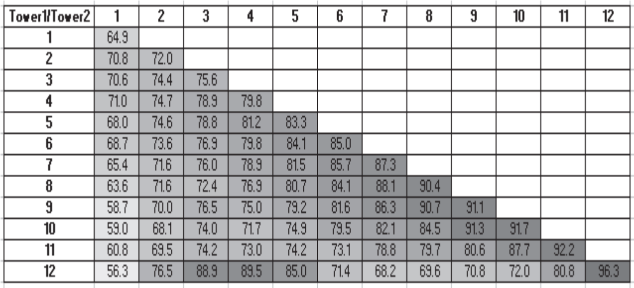
Story	Building1	Building2
6	1	
5	2	1
4	3	2
3	7	
2	9	
1	11	

Table 3 shows the example of consideration of carry weight of i-th story W_i of twin towers lumped mass model in Fig.4b.

In the same way to general shape building, Modification ratio (mr) from equation (11) is indicated to modify natural period T_{eigen} of every j mode of vibration by equation (12).

In order to evaluate the safety percentage in case of twin towers building, we use the same method as we used in case of general shape building.

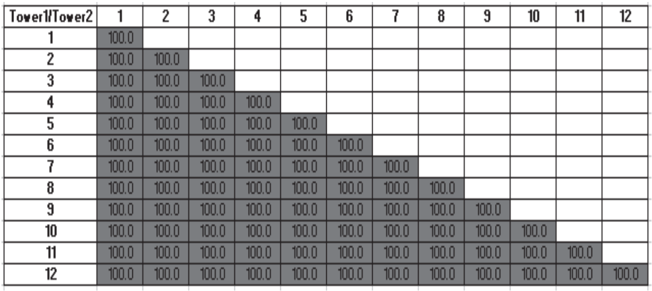
Table 4 Average of safety percentage of twin tower in case A (unit: percentage)



Tower1/Tower2	1	2	3	4	5	6	7	8	9	10	11	12
1	64.9											
2	70.8	72.0										
3	70.6	74.4	75.6									
4	71.0	74.7	78.9	79.8								
5	68.0	74.6	78.8	81.2	83.3							
6	68.7	73.6	76.9	79.8	84.1	85.0						
7	65.4	71.6	76.0	78.9	81.5	85.7	87.3					
8	63.6	71.6	72.4	76.9	80.7	84.1	88.1	90.4				
9	58.7	70.0	76.5	75.0	79.2	81.6	86.3	90.7	91.1			
10	59.0	68.1	74.0	71.7	74.9	79.5	82.1	84.5	91.3	91.7		
11	60.8	69.5	74.2	73.0	74.2	73.1	78.8	79.7	80.6	87.7	92.2	
12	56.3	76.5	88.9	89.5	85.0	71.4	68.2	69.6	70.8	72.0	80.8	96.3

Table 4 shows average of safety percentage of twin tower in case A of deflection of column in term of number of story of tower1 and tower2.

Table 5 Average of safety percentage of twin tower in case D (unit: percentage)



Tower1/Tower2	1	2	3	4	5	6	7	8	9	10	11	12
1	100.0											
2	100.0	100.0										
3	100.0	100.0	100.0									
4	100.0	100.0	100.0	100.0								
5	100.0	100.0	100.0	100.0	100.0							
6	100.0	100.0	100.0	100.0	100.0	100.0						
7	100.0	100.0	100.0	100.0	100.0	100.0	100.0					
8	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0				
9	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0			
10	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0		
11	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	
12	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

From the research, we found that $A_i$simple equation (3) is safe in order to determine seismic shear coefficient A_i in case of twin towers building by using the condition of deflection by case D as shown in Table 5.

3.4 *Axial force in rigid-beam analyzing method*

Generally, axial force in beams is not considered in process of structural design of general shape of building. On the other hand, in case of twin tower, different direction of movement of each tower is a cause of axial force in rigid-beams, which connect building A and building B such as the beam in Fig.4a. Therefore, axial force in rigid-beam due to movement of each tower must be considered in process of beam design. This research will show the axial force only in the top story of basement zone of twin tower because it will give a maximum value due to the movement. From the research, we have found that, effective mass value in the first mode is large and those in the second and third mode are small.

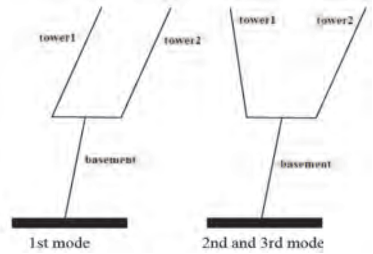


Fig.5 Vibrations mode of twin tower building

Figure 5 shows the first mode of vibration and the second and third mode of vibration.

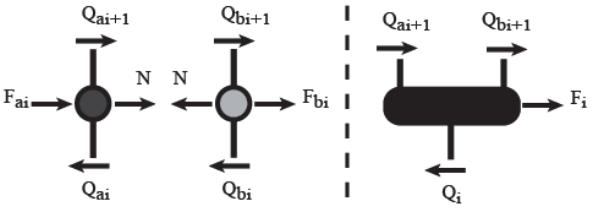


Fig.6 Free body diagram of connected story (1st mode of vibrations)

Figure 6 shows free body diagram of connected story to obtain axial force *N* in rigid-beam of first mode of vibrations. The directions of shear forces are determined by the figure of first mode. F_{ai}, F_{bi}, F_i show the inertia forces. Figure 6 (left) shows the part in Fig.4a. and figure 6 (right) shows the part in Fig.4. Both of the masses have been assumed to be equal and move under the same acceleration of ground motion. Then, we get that

$$F_{ai} = F_{bi} \ . \tag{13}$$

Equilibrium of shear force in i-th story gives

$$Q_{i(SRSS)} = Q_{ai(SRSS)} + Q_{bi(SRSS)}. \tag{14}$$

Assuming the ratio of $Q_{ai(SRSS)}$ and $Q_{bi(SRSS)}$ to relate to carry weights of each tower

$$Q_{bi(SRSS)} = Q_{i(SRSS)}/(\gamma+1) \tag{15}$$

$$Q_{ai(SRSS)} = \gamma Q_{i(SRSS)}/(\gamma+1) \tag{16}$$

where

$$\gamma = W_1/W_2 \tag{17}$$

W_1= carry weight of tower1 including top of basement story of building A

W_2= carry weight of tower2 including top of basement story of building B.

According to free body diagram in Fig.6

$$F_{ai} + N + Q_{ai+1(SRSS)} = Q_{ai(SRSS)} \tag{18}$$

$$F_{bi} - N + Q_{bi+1(SRSS)} = Q_{bi(SRSS)} \tag{19}$$

then

$$F_{ai} - F_{bi} + 2N + Q_{ai+1(SRSS)} - Q_{bi+1(SRSS)} = Q_{ai(SRSS)} - Q_{bi(SRSS)} \ . \tag{20}$$

Substituting eqns.(13) and (14) into equation (20) gives the following equation of the axial force in rigid beam occurred by the first mode of vibration.

$$N_1 = \frac{1}{2}\left(Q^{k=1}_{bi+1(SRSS)} - Q^{k=1}_{ai+1(SRSS)} + \frac{\gamma-1}{\gamma+1}Q_i\right) \ . \tag{21}$$

In this research, superscript k=1 means that we consider the number of considering mode is 1 to obtain the value $Q_{i(SRSS)}$.

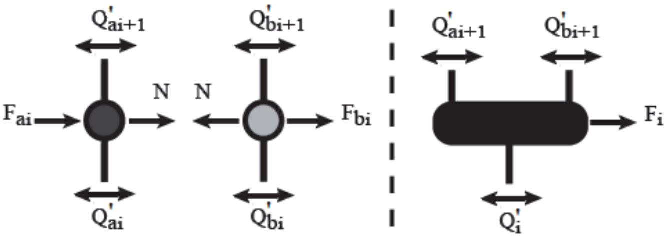


Fig.7 Free body diagram of connected story (2nd and 3rd mode of vibrations)

Figure 7 shows free body diagram of connected story to obtain axial force *N* in rigid-beam of second and third mode of vibrations. Dissimilar to the 1 first mode, we cannot know the movement direction of tower 1 and tower 2 in second and 3rd mode of vibrations, then we know that we must use the summation of $Q_{i(SRSS)}$ for safety condition in order to obtain the residual of axial force N_r in rigid-beam. It can be determine by the following equation.

$$N_r = \frac{1}{2}\left(Q'_{bi+1(SRSS)} + Q'_{ai+1(SRSS)} + \frac{\gamma-1}{\gamma+1}Q'_i\right) \tag{22}$$

where

$$Q'_{ai+1(SRSS)} = Q^{k=3}_{ai+1(SRSS)} - Q^{k=1}_{ai+1(SRSS)} \tag{23}$$

$$Q'_{bi+1(SRSS)} = Q^{k=3}_{bi+1(SRSS)} - Q^{k=1}_{bi+1(SRSS)} \ . \tag{24}$$

The safety value of axial force in rigid-beam *N* can be determined by the following equation

$$N = N_1 + N_r \ . \tag{25}$$

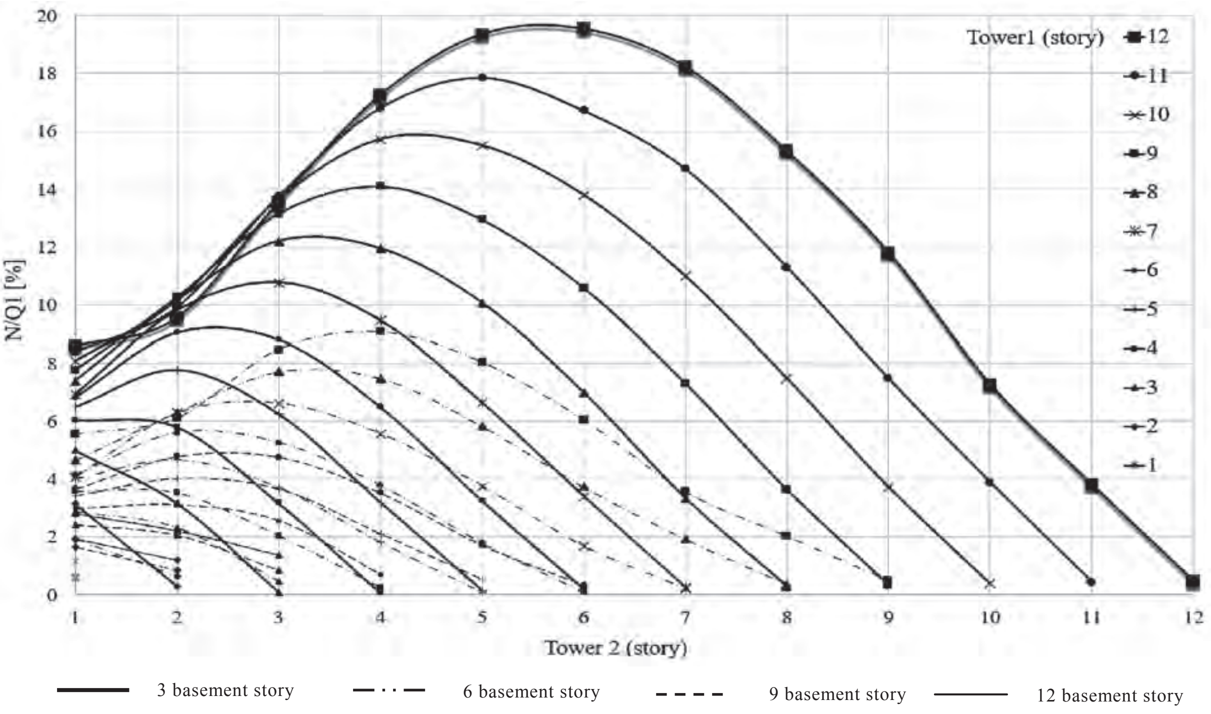


Fig.8 N/Q_1 in term of tower1, tower2 and basement story

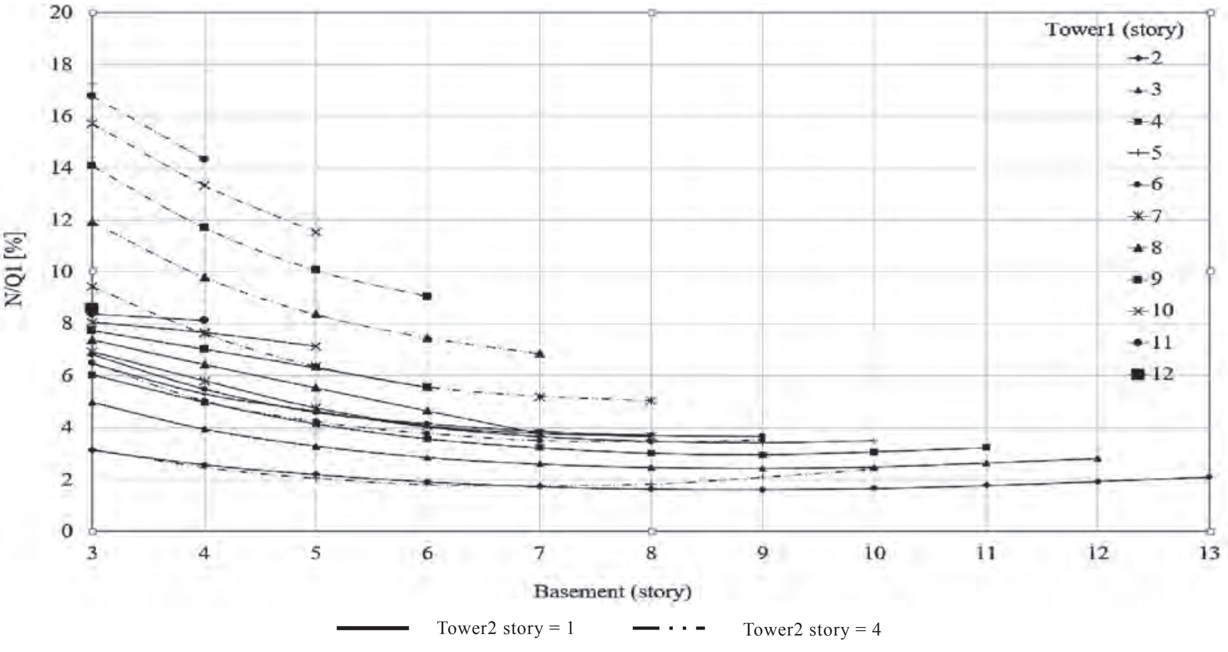


Fig.9 N/Q_1 in term of tower1, tower2 and basement story

Figure 8 shows the normalized axial force N/Q_1 in rigid beams in term of tower1, tower2, and basement story. From the research, the axial forces in rigid-beams close to zero when tower1 story and tower2 story are equal. According to Fig.9, when basement story are increased, the N/Q_1 decreases at the initial and it tends to be constant. Maximum axial force in rigid-beams as shown in Fig 8, usually occurs when tower2 story less than tower1 story about 5-6 stories. Furthermore, N/Q_1 remains constant in condition of number of story of tower2 is close to number of story of basement.

4. Conclusion

Design deformation angle of column is less or equal to 1/200 is usually used as a condition to design the earthquake resistant building in case of general shape building. But, in this paper, it is clarified that the “at least” condition cannot give safety seismic shear coefficient A_i as shown in case A, and the condition of design deformation angle of column should be as case D. (Design deformation angle of the first story’s column equals to 1/350, that of the top story’s column equals to 1/200 and those in between the first and top are linear interpolated value).

Next, we apply condition of case D in general shape building model to twin tower model. From the research, we found that, using combination of the following condition give safety seismic coefficient A_i in order to conduct the first phase design of earthquake resistant building in case of twin tower building.

1) Height h in natural period equation T must be the maximum height of building. This means that, we can use the same natural period T in all zone of building. For example, tower1, tower2, and basement zone.
2) In term of zoning of building, carry weight of i-th story W_i of tower1 and tower2 zone are separately calculated by its zone. For basement zone, it is calculated by including weight of tower1 zone, tower2 zone, and basement zone as shown in Table 3.

Furthermore, we found that, the axial forces in rigid-beams (connecting beam of building A and building B) close to zero when tower1 story and tower2 story are equal. Maximum axial force in rigid-beams usually occurs when tower2 story less than tower1 story about 5-6 stories.

Note that this paper considers seismic shear coefficient A_i due to the movement only in case 1 (both of towers move together in X direction) as shown in Fig.3. Therefore, in the future, it is necessary to study seismic shear coefficient A_i due to the movement in case 2 and case3 (both of towers move in Y direction).

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