Proc. Schl. Eng. Tokai Univ., Ser. E
39 (2014) 59-66

Effective Plate Thickness Range in Bending Technique for Levitation Control of Flexible Steel Plate

by

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(Received on Mar. 31, 2014 and accepted on July. 10, 2014)

Abstract

In a magnetic levitation, when a flexible and ultra-thin steel plate with a thickness of less than 0.3mm is to be levitated, levitation control becomes difficult because the thin steel plate undergoes increased flexure. We propose a levitation of an ultra-thin steel plate that is bent to an extent which does not induce plastic deformation. It has been confirmed that levitation performance is improved by bending a steel plate. In this study, to elucidate the effective plate thickness range of bending levitation, shapes of steel plates are analyzed using FDM and the relationship between average deflection and thickness of steel plate is examined. The results show that in the case of experiment conditions and steel plates, when levitating the steel plate with thickness thinner than 0.50 mm, the effect of bending levitation is remarkable.

Keywords: Magnetic levitation, Thin steel plate, Bending levitation, FDM

1. Introduction

Thin steel plates are widely used in automobiles, electrical appliances, cans and in other products. In recent years, it has become possible to manufacture ultra-thin steel plates, which are required in various fields with ever-increasing demands for surface quality. However, in the transport system in a thin-steel-plate production line, there is the problem that the quality of the plate surface deteriorates over time because the plate is usually in contact with rollers. To overcome this problem, studies of electromagnetic levitation technology have been carried out [1-4]. However, as the steel plate becomes thinner, the vibration caused by minute unpredictable factors, including the nonlinearity of the attractive force of the electromagnet and the change in resistance due to heat generation by the electromagnet, makes it difficult to maintain the levitation state. We propose a levitation of an ultra-thin steel plate that is bent to an extent that does not induce plastic deformation [5,6]. It has been confirmed that levitation performance is improved when a steel plate is bent and levitated [7]. However, the reason why the levitation of the steel plate is improved is not clear and the effective plate thickness range of bending levitation remains incompletely understood.

This paper is directed to a size of the steel plate that we use (length a = 800 mm, width b = 600 mm) through levitation experiment so far. In order to elucidate the effective plate thickness range of bending levitation, shapes of steel plates are analyzed using finite difference method (FDM) and average deflection quantities during the levitation are compared when steel plates are bent and when they are not bent. We examine the relationship between average deflection and thickness of steel plate and verify the effective plate thickness range of bending levitation.

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2. System for Control Experiment

Figure 1 shows a schematic illustration of experimental apparatus. Figure 2 shows an outline of the control system. The object of electromagnetic levitation is a rectangular zinc-coated steel plate (SS400) with length a = 800 mm, width b = 600 mm. To accomplish noncontact support of a rectangular thin steel plate using 5 pairs of electromagnets (No. 1-5) as if the plate was

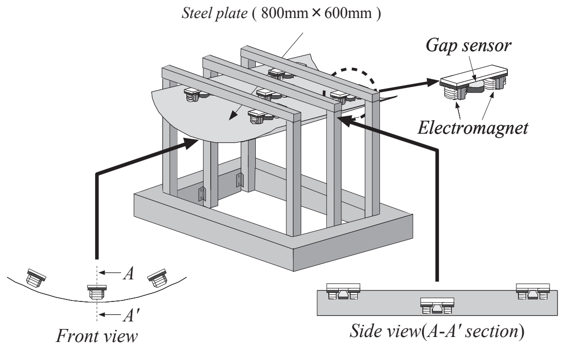


Fig. 1 Schematic illustration of experimental apparatus

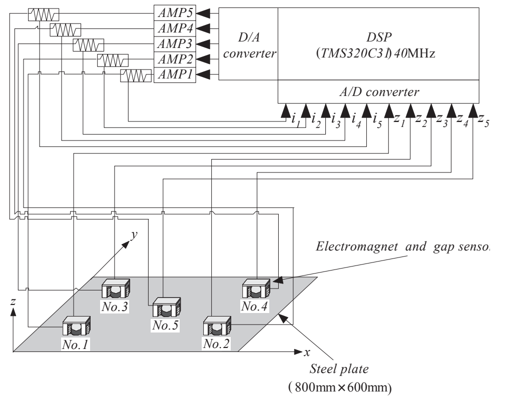


Fig. 2 Electromagnetic levitation control system

hoisted by strings, the displacement of the steel plate is measured by five eddy-current gap sensors. Here, the electric circuits of paired electromagnets are connected in series, while an eddy-current gap sensor is positioned between the two magnets of each pair. The detected displacement is converted to velocity using digital differentiation. In addition, the current in the coil of the electromagnets is calculated from the measured external resistance. Thus, obtained 15 measured values are input into the digital signal processor (DSP) via an A/D converter to calculate the control law. A control voltage is output from the D/A converter into a current-supply amplifier to control the attractive force of the 5 pairs of electromagnets in order that the steel plate is levitated below the surface of the electromagnets by 5 mm.

Among the 5 pairs of electromagnets, the 4 pairs at the corners can be inclined, as shown in the front view in Fig. 1. In addition, a central electromagnet can be moved in vertical direction. Thus, by moving 5 electromagnets, bending electromagnetic levitation of the steel plate is carried out.

Figure 3 shows the arrangement of the electromagnets. Regarding electromagnets arrangement, it determined as follows. Electromagnets arrangement of the long direction (x-axis) are positioned where the attractive force which supports the steel plate is the same when setting the electromagnets tilt angle as 0 degree. Also, electromagnets arrangements of the short direction (y-axis) are positioned where the whole deflection of steel plate becomes the smallest when steel plate is bent and levitated. Due to offset

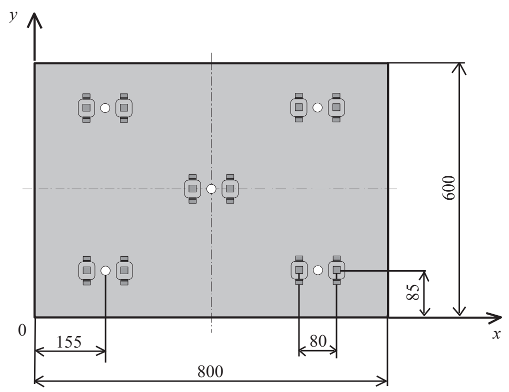


Fig. 3 Arrangement of electromagnets

caused by the bending, the positions of the outside electromagnets (No. 1-4) are adjusted so that the attractive force applied according to their original fixed positions on the thin steel plate even when it is bent. As a tilt angle of electromagnets becomes large, restorative force of the steel plate increases. Therefore, the attractive forces of outside electromagnets become large and central electromagnet force becomes small with the increase in tilt angle. Finally, steel plate is supported with 4 pairs of outside electromagnets. The attractive forces applied to each electromagnet when bending a steel plate are calculated by considering a steel plate to be a beam and computed from the relationship between the bending angle of a supporting point and the reactive force.

In order to compute the shape of steel plate using FDM, we assume that there is levitation without bending when the steel plate is supported with 5 electromagnets, and levitation with bending when the steel plate is supported with 4 electromagnets. By these assumptions, deflection analysis is conducted.

3. Finite Difference Analysis

3.1 *Analysis model*

The displacement of the rectangular thin steel plate subjected to gravity and shape of steel plate while being levitated are calculated. The equations for the static displacement of the rectangular thin steel plate are expressed as

$$D\nabla^4 z = \rho h g \tag{1}$$

$$D = \frac{Eh^3}{12(1-\nu^2)}, \ \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

where E is the Young's modulus of the thin steel plate [N/m^2], h is the plate thickness [m], ν is the Poisson ratio, x and y are the coordinates in the long and short directions [m], respectively, z is the vertical displacement of the plate [m], ρ

is the plate density [kg/m^3], and g is the acceleration due to gravity [m/s^2].

Difference approximation of the each clause of Eq. (1) appears as follows.

$$\frac{\partial^4 z}{\partial x^4} = \frac{z_{i+2,j} - 4z_{i+1,j} + 6z_{i,j} - 4z_{i-1,j} + z_{i-2,j}}{\Delta x^4} \tag{2}$$

$$\frac{\partial^4 z}{\partial y^4} = \frac{z_{i,j+2} - 4z_{i,j+1} + 6z_{i,j} - 4z_{i,j-1} + z_{i,j-2}}{\Delta y^4} \tag{3}$$

$$\frac{\partial^4 z}{\partial x^2 \partial y^2} = \frac{z_{i+1,j+1} - 2z_{i,j+1} + z_{i-1,j+1}}{\Delta x^2 \Delta y^2} + \frac{-2z_{i+1,j} + 4z_{i,j} - 2z_{i-1,j}}{\Delta x^2 \Delta y^2} + \frac{z_{i+1,j-1} - 2z_{i,j-1} + z_{i-1,j-1}}{\Delta x^2 \Delta y^2} \tag{4}$$

Δx : Calculated intervals of long direction of the thin steel plate (x-axis) [m]

Δy : Calculated intervals of short direction of the thin steel plate (y-axis) [m]

i : Calculated number for the long direction of the thin steel plate (x-axis)

j : Calculated number for the short direction of the thin steel plate (y-axis)

Here, collating Eqs. (2) ~ (4) with $\Delta x = \Delta y = \lambda$, it will be shown as follows.

$$\nabla^4 z = \frac{1}{\lambda^4}\{z_{i+2,j} + z_{i-2,j} + z_{i,j+2} + z_{i,j-2} + 2(z_{i+1,j-1} + z_{i+1,j+1} + z_{i-1,j-1} + z_{i-1,j+1}) - 8(z_{i+1,j} + z_{i-1,j} + z_{i,j+1} + z_{i,j-1}) + 20z_{i,j}\} \tag{5}$$

Equation (5) shows the connection between the calculated numbers i and j. These calculated numbers i and j are taken as 1 for coordinates 0 mm of both the long and short directions and the number increases by 1 at each interval. In addition, the calculated number is considered as $i = p$ with long distance coordinate x = 800 mm and as $j = q$ with short distance coordinate y = 600 mm. From this, deflection formulas of the thin steel plate are made for each calculated point. These are collated and deflection analysis is carried out. Here, we will consider the corners of (x, y) = (0 mm, 0 mm). At this point, difference approximation of Eq. (5) appears as follows.

$$\nabla^4 z = \frac{1}{\lambda^4}\{z_{3,1} + z_{-1,1} + z_{1,3} + z_{1,-1} + 2(z_{2,0} + z_{2,2} + z_{0,0} + z_{0,2}) - 8(z_{2,1} + z_{0,1} + z_{1,2} + z_{1,0}) + 20z_{1,1} + 2(z_{2,0} + z_{2,2} + z_{0,0} + z_{0,2}) - 8(z_{2,1} + z_{0,1} + z_{1,2} + z_{1,0}) + 20z_{1,1}\} \tag{6}$$

Here, $z_{0,0}$ and $z_{-1,1}$ etc, which exist in Eq. (6), the corners and surrounding area of the thin steel plate are virtual points beyond the thin steel plate. This virtual point is replaced by points existing in accordance with boundary conditions. The derivation of the representative virtual point is shown below.

Since the boundary conditions in the free edge (x = 0 mm, 800 mm) have a bending moment of zero, the boundary conditions are expressed as follows.

$$\frac{\partial^2 z}{\partial y^2} + \nu \frac{\partial^2 z}{\partial x^2} = 0 \tag{7}$$

The equivalent sheering power combined the twisting moment and sheering force is zero.

$$\frac{\partial^3 z}{\partial y^3} + (2-\nu)\frac{\partial^3 z}{\partial y \partial x^2} = 0 \tag{8}$$

The boundary conditions in the free edge (y = 0 mm, 600 mm) have a bending moment of zero.

$$\frac{\partial^2 z}{\partial x^2} + \nu \frac{\partial^2 z}{\partial y^2} = 0 \tag{9}$$

The equivalent sheering power is zero.

$$\frac{\partial^3 z}{\partial x^3} + (2-\nu)\frac{\partial^3 z}{\partial x \partial y^2} = 0 \tag{10}$$

In addition, the corners in the free edge (x, y) = (0 mm, 0 mm), (0 mm, 600 mm), (800 mm, 600 mm), (800 mm, 0mm) have the twisting moments of zero.

$$\frac{\partial^2 z}{\partial x \partial y} = 0 \tag{11}$$

The boundary conditions of Eqs. (7) ~ (11) are expressed with difference approximations and instances of difference approximations in $\nabla^4 z$ are shown in the appendix (Eqs. (A1) ~ (A26)).

Equations (A18) ~ (A26) in the appendix are substituted for Eq. (1), arranged and expressed in matrix form as shown below.

$$\frac{D}{\lambda^4}\boldsymbol{AZ} = \rho hg\boldsymbol{E} \tag{12}$$

Equation (12) is arranged as shown below.

$$\boldsymbol{Z} = \frac{\lambda^4}{D}\rho hg\boldsymbol{A}^{-1}\boldsymbol{E} \tag{13}$$

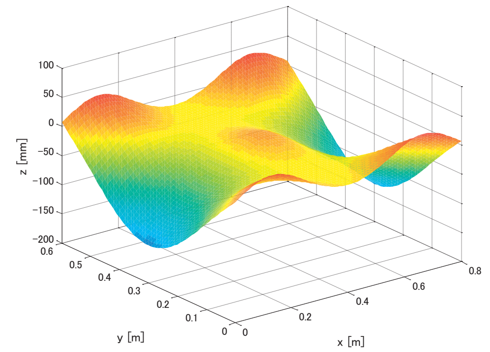
$\boldsymbol{Z}$: Discretized displacement of steel plate

$\boldsymbol{A}$: Difference coefficient matrix

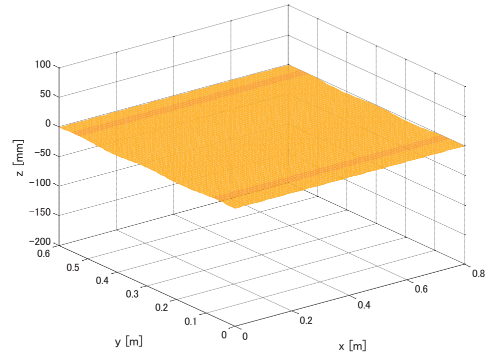
$\boldsymbol{E}$: $(1\ 1\ 1\ \cdots\ 1\ 1)^{\mathrm{T}}$

Using eq. (13), the displacement of the thin steel plate is calculated by the FDM. We calculate that the steel plate is simply supported at the position of the electromagnets. The size of FDM mesh is 10 mm × 10 mm.

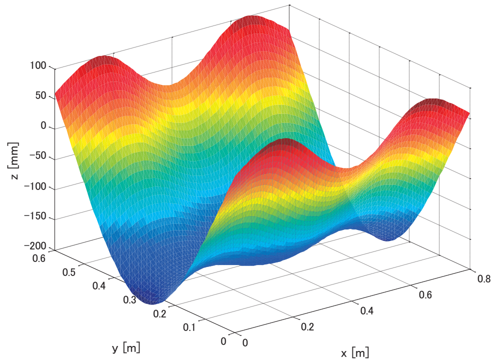
The shape of the steel plates during the levitation is evaluated and it is necessary to investigate the plate thickness which bending levitation is effective. However, rigidity differs depending on the plate thickness and direct comparison with deflection quantity is therefore not possible. Thus, the shapes of the steel plates shown in Figs. 4 (b) and (d), which restrict the change of the long direction of the same steel plate are taken as standard and the deflection



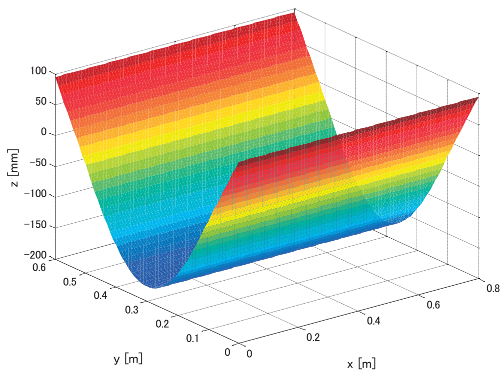
(a) Analysis result of FDM (5 points support)



(b) Reference plane (5 points support)

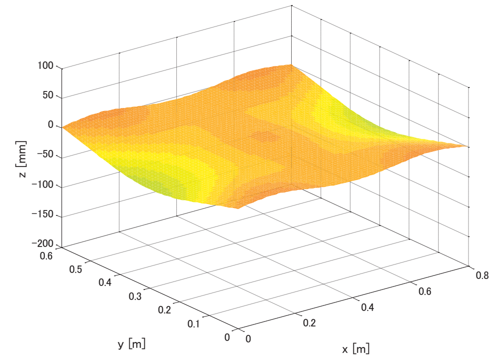


(c) Analysis result of FDM (4 points support)

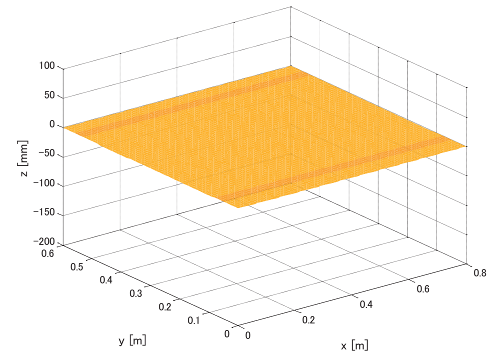


(d) Reference plane (4 points support)

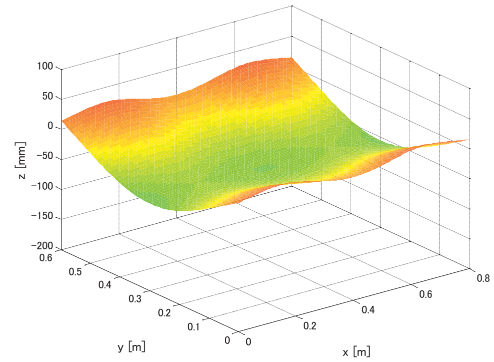
Fig. 4 Example of steel plate's shape
(Plate thickness h = 0.1 mm)



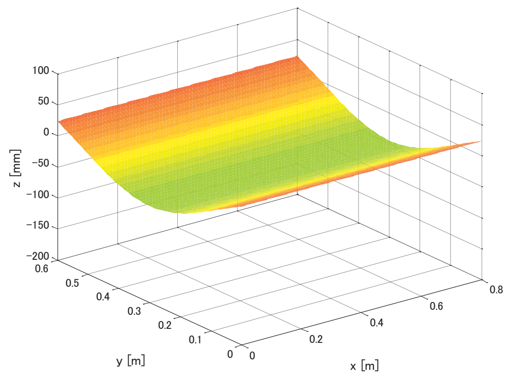
(a) Analysis result of FDM (5 points support)



(b) Reference plane (5 points support)



(c) Analysis result of FDM (4 points support)



(d) Reference plane (4 points support)

Fig. 5 Example of steel plate's shape
(Plate thickness h = 0.2 mm)

quantity from the reference plane is evaluated. As examples of the analysis results, the shapes during the levitation of thin steel plates with a thickness of 0.1 mm and 0.2 mm, a length of 800 mm and a width of 600 mm are shown in Fig. 4 and Fig. 5. Evaluation value J is defined as shown below.

$$J = \frac{\overline{Z}_5 - \overline{Z}_4}{h} \tag{14}$$

$$\overline{Z}_5 = \frac{\sum_{i=1}^{N} |Z_{5i} - Z_{05}|}{N} \tag{15}$$

$$\overline{Z}_4 = \frac{\sum_{i=1}^{N} |Z_{4i} - Z_{04}|}{N} \tag{16}$$

$\overline{Z}_5$: Average deflection of steel plate (5 points support) [m]
$\overline{Z}_4$: Average deflection of steel plate (4 points support) [m]
Z_{5i}: Displacement at each analysis point on the thin steel plate (5 points support) [m]
Z_{05}: Reference plane (5 points support) [m]
Z_{4i}: Displacement at each analysis point on the thin steel plate (4 points support) [m]
Z_{04}: Reference plane (4 points support) [m]
N: Total number of analysis points (N = 4941)

J expresses the difference of the average deflection of 5 points support $\overline{Z}_5$ and 4 points support $\overline{Z}_4$ divided by plate thickness h. When levitating flexible steel plates, the deflection arises at the place where the magnetic attractive force is not impressed, leading to difficulty in levitation. Therefore, levitation performance is improved by suppressing the deflection of steel plate. In order to evaluate change of deflection by bending levitation, we set the average deflection from the reference plane supported by 5 points as $\overline{Z}_5$ and the average deflection from the reference plane supported by 4 points as $\overline{Z}_4$. Thus, the relationship between plate thickness h and evaluation value J is computed.

3.2 *Analysis results*

Figure 6 shows the relationship between plate thickness h, average deflection $\overline{Z}_5$ and $\overline{Z}_4$. Figure 7 shows the relationship between plate thickness h and evaluation value J. From Fig. 6, average deflection of steel plate supported by 4 points are smaller than 5 points. It is indicated that average deflection during the levitation decreases with the bending of steel plate. In addition, as the steel plate becomes thinner, the difference of the average deflection of 5 points support and 4 points support increases. It shows that average deflection during the levitation is reduced by bending a steel plate. Therefore, especially in the case of an ultra-thin steel plate, the reduction effect of average deflection is remarkable by bending a steel plate.

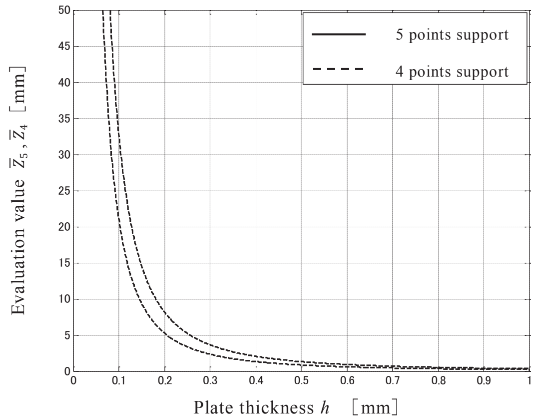


Fig. 6 Relationship between plate thickness h, $\overline{Z}_5$ and $\overline{Z}_4$

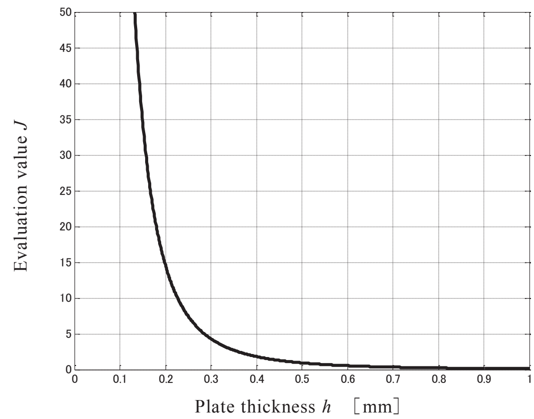


Fig. 7 Relationship between plate thickness h and J

Further, Fig. 7 shows that evaluation value J decreases as plate thickness increases and where plate thickness is 0.50 mm, J becomes less than 1. This means that the difference in average deflection of 4 and 5 points of support is smaller than plate thickness and change in average deflection due to bending levitation cannot be seen. Since average deflection hardly arises in the steel plate with a thickness of 0.50 mm or more, there is no necessity of carrying out bending levitation.

From these results, in the case of experiment conditions and steel plates, when levitating the steel plate with thickness thinner than 0.50 mm, the effect of bending levitation is remarkable. Through this analysis, it is possible to estimate the effective plate thickness range of bending levitation.

4. Conclusion

This paper presented the basic consideration on the bending levitation intended for the size of the steel plate which we used (length a = 800 mm, width b = 600 mm) in the levitation experiment. In order to elucidate the effective plate thickness range of bending levitation, shapes of steel plates during the levitation were analyzed using FDM and the relationship between average deflection and thickness of steel plate was examined. In the case of experiment conditions and steel plates, as it was confirmed that bending levitation effect appeared notably using a plate thickness of less than 0.50 mm, in an ultra-thin steel plate, the effect of bending levitation was remarkable.

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Appendix

In the free edge of x = 0 mm

$$z_{i,2}-2z_{i,1}+z_{i,0}+v(z_{i+1,1}-2z_{i,1}+z_{i-1,1})=0 \tag{A1}$$

$$z_{i,3}-2z_{i,2}+2z_{i,0}-z_{i,-1}+(2-v)(z_{i+1,2}-2z_{i,2}+z_{i-1,2}-z_{i+1,0}$$
$$+2z_{i,0}-z_{i-1,0})=0 \tag{A2}$$

However, $1 \leqq i \leqq p$

In the free edge of y = 0 mm

$$z_{0,j}-2z_{1,j}+z_{2,j}+v(z_{1,j+1}-2z_{1,j}+z_{1,j-1})=0 \tag{A3}$$

$$z_{-1,j}-2z_{0,j}+2z_{1,j}-z_{2,j}+(2-v)(z_{0,j+1}-2z_{0,j}+z_{0,j-1}$$
$$-z_{2,j+1}+2z_{2,j}-z_{2,j-1})=0 \tag{A4}$$

However, $1 \leqq j \leqq q$

Collating Equations (A1) ~ (A4) in the appendix, the value of the virtual point is found as shown below.

In the free edge of x = 0 mm

$$z_{i,0}=2z_{i,1}-z_{i,2}-v(z_{i+1,1}-2z_{i,1}+z_{i-1,1}) \tag{A5}$$

$$z_{i,-1}=(-v^2+2v)(z_{i+2,1}+z_{i-2,1})+(4v^2-8v-4)(z_{i+1,1}+z_{i-1,1})$$
$$+(-2v+4)(z_{i+1,2}+z_{i-1,2})+(-6v^2+12v+12)z_{i,1}$$
$$+(4v-12)z_{i,2}+z_{i,3} \tag{A6}$$

However, $3 \leqq i \leqq p$-2

In the free edge of y = 0 mm

$$z_{0,j}=+2z_{1,j}-z_{2,j}-v(z_{1,j+1}-2z_{1,j}+z_{1,j-1}) \tag{A7}$$

$$z_{p+2,j}=(-v^2+2v)(z_{1,j+2}+z_{1,j-2})+(4v^2-8v-4)(z_{1,j+1}$$
$$+z_{1,j-1})+(-2v+4)(z_{2,j+1}+z_{2,j-1})+(-6v^2+12v$$
$$+12)z_{1,j}+(4v-12)z_{i,q-1}+z_{i,q-2} \tag{A8}$$

However, $3 \leqq j \leqq q$-2

In the corner of (x, y) = (0 mm, 0 mm)

$$z_{2,0}=2z_{2,1}-z_{2,2}-v(z_{1,1}-2z_{2,1}+z_{3,1}) \tag{A9}$$

$$z_{2,-1}=(2v^2-4v-4)z_{1,1}+(-5v^2+10v+12)z_{2,1}+(4v^2-8v-4)z_{3,1}$$
$$+(-v^2+2v)z_{4,1}+(-2v+4)(z_{1,2}+z_{3,2})$$
$$+(4v-12)z_{2,2}+z_{2,3} \tag{A10}$$

$$z_{1,0}=2z_{1,1}-z_{1,2} \tag{A11}$$

$$z_{1,-1}=(-2v^2+12)z_{1,1}+(4v^2-4v-8)z_{2,1}+(-2v^2+4v)z_{3,1}$$
$$+(4v-12)z_{1,2}+(-4v+8)z_{2,2}+z_{1,3} \tag{A12}$$

$$z_{0,0}=2z_{2,1}+2z_{1,2}-3z_{2,2}-v(2z_{1,1}-2z_{2,1}+z_{3,1}-2z_{1,2}+z_{1,3}) \tag{A13}$$

$$z_{0,1}=2z_{1,1}-z_{2,1} \tag{A14}$$

$$z_{-1,1}=(-2v^2+12)z_{1,1}+(4v^2-12)z_{2,1}+z_{3,1}+(4v^2-4v-8)z_{1,2}$$

$$+(-4v+8)z_{2,2}+(-2v^2+4v)z_{1,3} \quad \text{(A15)}$$

$$z_{0,2}=2z_{1,2}-z_{2,2}-v(z_{1,1}-2z_{1,2}+z_{1,3}) \quad \text{(A16)}$$

$$z_{-1,2}=(2v^2-4v-4)z_{1,1}+(-5v^2+10v+12)z_{1,2}$$
$$+\{4v^2-8v-4\}z_{1,3}+(-v^2+2v)z_{1,4}$$
$$+(-2v+4)(z_{2,1}+z_{2,3})+(4v-12)z_{2,2}+z_{3,2} \quad \text{(A17)}$$

Also, deriving the values of the virtual points about the corners and surrounding area in the free edge of x = 800 mm and y = 600 mm is omitted due to limitations of space.

Here, difference approximation of $\nabla^4 z$ of the deflection formula sequentially from i = 1, j = 1, and substitution of the virtual point in equations (A5) ~ (A17) is written as shown below.

In the case of i = 1, j = 1

$$\nabla^4 z_{1,1}=\frac{1}{\lambda^4}\{(-2v^2+2)(z_{3,1}+z_{1,3})+(4v^2+8v-12)z_{2,1}$$
$$+(-4v^2-8v+12)z_{1,1}+(-8v+8)z_{2,2}+(4v^2+8v-12)z_{1,2}\}$$
$$\text{(A18)}$$

In the case of i = 2, j = 1

$$\nabla^4 z_{2,1}=\frac{1}{\lambda^4}\{(-v^2+1)z_{4,1}+(4v^2+4v-8)z_{3,1}$$
$$+(-5v^2-8v+15)z_{2,1}+(2v^2+4v-6)z_{1,1}$$
$$+(-2v+4)(z_{3,2}+z_{1,2})+(4v-12)z_{2,2}+2z_{2,3}\} \quad \text{(A19)}$$

In the case of $3 \leqq i \leqq p$-2, j = 1

$$\nabla^4 z_{i,1}=\frac{1}{\lambda^4}\{(-v^2+1)(z_{i-2,1}+z_{i+2,1})$$
$$+(4v^2+4v-8)(z_{i-1,1}+z_{i+1,1})+(-6v^2-8v+16)z_{i,1}$$
$$+(-2v+4)(z_{i-1,2}+z_{i+1,2})+(4v-12)z_{i,2}+2z_{i,3}\} \quad \text{(A20)}$$

In the case of i = 1, j = 2

$$\nabla^4 z_{1,2}=\frac{1}{\lambda^4}\{(-2v+4)(z_{2,1}+z_{2,3})+(2v^2+4v-6)z_{1,1}+2z_{3,2}$$
$$+(4v-12)z_{2,2}+(-5v^2-8v+15)z_{1,2}$$
$$+(4v^2+4v-8)z_{1,3}+(-v^2+1)z_{1,4}\} \quad \text{(A21)}$$

In the case of i = 2, j = 2

$$\nabla^4 z_{2,2}=\frac{1}{\lambda^4}\{(-2v+2)z_{1,1}+(2v-6)(z_{2,1}+z_{1,2})$$
$$+(-v+2)(z_{3,1}+z_{1,3})+2z_{3,3}+z_{4,2}+z_{2,4}$$
$$-8(z_{3,2}+z_{2,3})+18z_{2,2}\} \quad \text{(A22)}$$

In the case of $3 \leqq i \leqq p$-2, j = 2

$$\nabla^4 z_{i,2}=\frac{1}{\lambda^4}\{(-v+2)(z_{i-1,1}+z_{i+1,1})+(2v-6)z_{i,1}$$
$$+z_{i-2,2}+z_{i+2,2}+z_{i,4}-8(z_{i-1,2}+z_{i+1,2}+z_{i,3})$$
$$+19z_{i,2}+2(z_{i-1,3}+z_{i+1,3})\} \quad \text{(A23)}$$

In the case of i = 1, $3 \leqq j \leqq q$-2

$$\nabla^4 z_{1,j}=\frac{1}{\lambda^4}\{(-v^2+1)(z_{1,j-2}+z_{1,j+2})$$
$$+(-2v+4)(z_{2,j-1}+z_{2,j+1})+(4v^2+4v-8)(z_{1,j-1}+z_{1,j+1})$$
$$+2z_{3,j}+(4v-12)z_{2,j}+(-6v^2-8v+16)z_{1,j}\} \quad \text{(A24)}$$

In the case of i = 2, $3 \leqq j \leqq q$-2

$$\nabla^4 z_{2,j}=\frac{1}{\lambda^4}\{(-v+2)(z_{1,j+1}+z_{1,j-1})+(2v-6)z_{1,j}$$
$$+z_{4,j}+z_{2,j+2}+z_{2,j-2}+2(z_{3,j-1}+z_{3,j+1})$$
$$-8(z_{3,j}+z_{2,j+1}+z_{2,j-1})+19z_{2,j}\} \quad \text{(A25)}$$

In the case of $3 \leqq i \leqq p$-2, $3 \leqq j \leqq q$-2

$$\nabla^4 z_{i,j}=\frac{1}{\lambda^4}\{z_{i+2,j}+z_{i-2,j}+z_{i,j+2}+z_{i,j-2}+2(z_{i+1,j-1}$$
$$+z_{i+1,j+1}+z_{i-1,j-1}+z_{i-1,j+1})-8(z_{i+1,j}+z_{i-1,j}$$
$$+z_{i,j+1}+z_{i,j-1})+20z_{i,j}\} \quad \text{(A26)}$$