

# Demand and Price Forecasting Models for Strategic and Planning Decisions in a Supply Chain

by

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## Abstract

Procurement and sales forecasting are a fundamental approach to the estimation of inventory. The accuracy is an essential factor for a firm on cost of manufacturing. Forecasting is an important for all strategic and planning decisions in a supply chain. Forecasts of product demand, materials, labor, financing are useful inputs to scheduling, acquiring resources, and determining resource requirement. However, highly volatile and correlated demand caused inaccuracy. This paper aims to compare real option method and ARIMA model method to be able to predict a precise future procurement and sales. To validate the forecasting model, we use the real world data which are demand obtained from one die casting company and copper price. The result shows that real option model is outperforming ARIMA model quite significantly in case the data has high volatility.

**Keywords:** Forecast, Volatile, Demand, Geometric Brownian motion, ARIMA methodology

## 1. Introduction

Smoothing the demand and supply is the main problem in supply chain management. Under supply leads to the loss of potential revenue, over supply incurs extra carrying cost of the excess product. Procurement and inventory control are of active area of research to gain the competitiveness for the company. The optimal quantity can enable the company to achieve the cost reduction. To obtain the optimality in supply chain, we have to deal with making the decision about the future event, produce to secure the future sale. This involves with forecasting the unforeseen data for example demand and price. The less deviation from the real value the less risk we have from doing the operation.

Demand forecasting plays a very crucial role in production planning of the manufacturing. It is the main input for the capacity planning which links to various chain of supply including inventory, logistic and service level.

Demand and price is usually difficult to forecast due to their uncertainty nature. Basically demand should show an increasing trend over the long term period. When we look at the short term period, we often see the unexpected move either going up or down. If we include more realistic possibility of the economic factor to try to understand and

predict movement of the demand, we can clearly see from the history that will not always suppose to have an upward trend even in the long term demand. This case is clearly noticed in the time of economic crisis. Besides, competition and advancement of technology also erode the demand of one company to another. Taking the example of mobile music industry like Sony that loses the market share to Apple product.

In this paper we compare the new forecasting method that is apply to the forecast of demand and price. There are many ways that we can adopt for forecasting for example linear regression, moving average, ARMA, ARIMA. Here we compare the real option approach with the ARIMA model and conclude what kind of data is best suited for these kind of forecasting.

Real option approach is originated from the finance study. It is used to value the value of uncertainty in an investment. Real option is different from traditional investment criteria like Net Present Value (NPV) in that unexercised choices have also future value and should contribute the evaluation.

## 2. Literature review

Real option approaches which is based on the diffusion process of the variable for doing the forecast is quite a new field of study. The most common diffusion process that is used in real option study is Geometric Brownian Motion

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(GBM). GBM is used to replicate the movement or character of the object of study in many fields of study e.g. particle movement in physics, the price of the underlying asset in the financial study, water level in reservoir etc. Huang [1] applies real option in forecasting the demand of products that has correlated to one another and that have high volatility. The kind of product that has such kind of nature is referred to IT product. In their research they forecast the demands of 4 products that relate to one another. If one product changes, the volume of other products also changes.

The GBM model is used to replicate the future demand in capacity studies [2]. They studied capacity utilization over time assuming that demand followed a GBM. Yon [3] also studied about the capacity planning. They analyzed in case of demand has high volatility and machine and process technology are advanced at a rapid pace, making it difficult to do the capacity planning in the semiconductor industry. They assume the demand to follow the GMB. They simulate and compare the result of the profit in two difference scenarios.

Real option by mean of GBM has also been used extensively in making investment decision as in Dixit and Pindyck [4]. It can be used to find the optimal time to invest, evaluate option to postpone investment, option to shut down the business, option to expand business etc. in which the traditional investment criteria cannot handle with well.

An autoregressive integrated moving average (ARIMA) is one of well known model in statistic and econometrics. It is used to study the behavior of time series and predict the future out of the time series. The simple way to think about the ARIMA model is that it is the combination of random walk and trend model. This characteristic is very similar to the GBM and it is a good candidate for the comparison.

For a high volatile demand product, Makridakis and Wheelwright [5] extended the time series analysis by including moving average, regression, exponential smoothing, and seasonal exponential smoothing. The ARIMA methodology is developed by Box and Jenkins [6]. It is a mathematical model for forecasting a time series by fitting it with the data and using the fitted model for forecasting. R.J. Hyndman [7], Pankratz [8] and Martin, R.D., Samarov, A. and W. Vandaele [9] study ARIMA model in application.

Metcalf & Hasset [10] study the mean reversion format equation, and it has more economic logic than the geometric Brownian model. Carlos and David [11] explore the main stochastic price processes used to model energy spot and forward prices for derivatives valuation and risk management. They apply a Geometric Brownian motion to be a random walk with mean reversion. In our research we compare the forecasting methods which are real option method, ARIMA

model for the estimation of procurement and sales. We compare our study with the previous study in Table 1.

**Table 1 Previous study comparison**

Previous Study	Details
Makridakis and Wheelwright (1985)	Time series analysis by including moving average, regression, exponential smoothing, and seasonal exponential smoothing
Ming-Guan Huang (2008)	Real option approach-based demand forecasting method for a range of products with highly volatile and correlated demand
James D. Hamilton (Book, 1994)	Analyze vector auto regressions, generalized method of moments, the economic and statistical consequences of unit roots, time-varying variances, and nonlinear time series models

The outline of this study is as follow: In section 2 we explain the methodologies and how to estimate the future value. We set a numerical simulation and illustrate results in section 3. We prefer to use MATLAB to simulate the simulation of real option method, Eviews for ARIMA model. Finally, we conclude a summary of our findings.

### 3. Forecasting Methodology

#### 3.1 Geometric Brownian motion

Geometric Brownian motion is originated in physics as a diffusion process used to describe the partial motion when it falls. Let  $X_t$  be the value of the data at time  $t$  and assume it to follow the Geometric Brownian Motion, we have the diffusion specify in equation (1). In this equation there are two parts. The first part on the right hand side is the mean growth of the diffusion and the second part on the right hand side of the equation is the volatility.

$$dX_t = \mu X_t dt + \sigma X_t dz_t \quad (1)$$

Where  $dz_t = \varepsilon_t \sqrt{dt}$  is the standard increment of Wiener process or the white noise,  $\mu$  and  $\sigma$  are drift and variance terms respectively.  $z_t$  has the following 2 properties:

**Property 1:** The change in the value of  $z$ ,  $dz$ , over a time interval of length  $\Delta t$  is proportional to the square root of  $\Delta t$

where the multiplier is random. In specific  $dz=z(t+\Delta t)-z(t)=\varepsilon\Delta t$ , where  $\varepsilon$  is a standard normal random variable. Hence values of  $dz$  follow a normal distribution with mean 0 and variance equal to the change in time ( $\Delta t$ ) over which  $dz$  is measured.

**Property 2:** The changes in the value of  $z_t$  for any non-overlapping intervals of time are independent.

Let  $f=\ln X_t$  and expand  $f$  according to the Taylor series we have

$$df = \frac{df}{dX_t}(dX_t) + \frac{1}{2} \frac{d^2f}{dX_t^2}(dX_t)^2 + \dots \quad (2)$$

Then apply Ito lemma to (2), we have that  $\Delta t$  that has power greater than 2 will approach to zeros quicker when  $\Delta t$  is small. We have

$$df = \frac{df}{dX_t}(dX_t) + \frac{1}{2} \frac{d^2f}{dX_t^2}(dX_t)^2 \quad (3)$$

Substitute (1) into (3) and rearrange we have

$$\ln X_{t+1} - \ln X_t = \left( \mu - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma \varepsilon \sqrt{\Delta t} \quad (4)$$

$$X_{t+1} = X_t \exp \left[ \left( \mu - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma \varepsilon \sqrt{\Delta t} \right] \quad (5)$$

Equation (5) is the discrete version of Geometric Brownian Motion that we will use to simulate the forecast of demand and price in this study.

### 3.2 Calibrate the parameter

The Geometric Brownian Motion process can formally be defined as  $X_t, 0 \leq t \leq \infty$ , the random variable of  $X_t/X_{t+1}$  is independent of all values of the variable up to the time  $t$  and if in addition  $\ln(X_t/X_{t+1})$  has a normal distribution with mean  $\mu t$  and variance  $\sigma^2 t$ . Let  $r_t$  be the difference between logarithm  $X_t$  and  $X_{t-1}$  we have

$$\ln \left( \frac{X_t}{X_{t-1}} \right) = r_t \quad (6)$$

In finance, (6) represent the logarithm of the return which is necessary assumption because the value of the underlying asset that is assumed to follow Geometric Brownian Motion should have a value greater or equal to zero.  $r_t$  also implies the growth rate of the observed data in each period.

In order to estimate the parameter to use in the forecast model as specified by equation (5) we following equations

$$E(r_t) = (\mu - \sigma^2/2)\Delta t \quad (7)$$

$$V(r_t) = \sigma^2 \Delta t \quad (8)$$

$$\bar{r} = \frac{\sum_{t=1}^n r_t}{n} \quad (9)$$

$$s_r = \sqrt{\frac{1}{(n-1)} \sum_{t=1}^n (r_t - \bar{r})^2} \quad (10)$$

$$\sigma = \frac{s_r}{\Delta t} \quad (11)$$

$$\mu = \frac{\bar{r}}{\Delta t} + \frac{\hat{\sigma}^2}{2} = \frac{\bar{r}}{\Delta t} + \frac{s_r^2}{2\Delta t} \quad (12)$$

### 3.3 ARIMA Model

In supply chain management, An autoregressive integrated moving average has been used to in many problems ranging from finding optimal order policy, predicting customer demand, predicting the price movement, inventory management etc.

The autoregressive moving average (ARMA) model was introduced by Box and Jenkins (1976). It consists of two parts, an autoregressive (AR) part and a moving average (MA) part. Specifically, the three types of parameters in the model are: the autoregressive parameters (p), the number of differencing passes (d), and moving average parameters (q). Models are summarized as ARIMA (p, d, q).

ARIMA combines auto regression-which fits the current data point to a linear function of some prior data points and moving averages adding together several consecutive data points and getting their mean, and then using that to compute estimations of the next value. With enough elements regressed and averaged, we can fit an approximation too many time series and with low discrepancy.

We find the best ARIMA (p,d,q) model condition by comparing the Akaike information criterion (AIC). The smallest AIC is the most suitable condition. An ARIMA (p,d,q) model contains three different kinds of parameters: (1) the p AR-parameters; (2) the q MA-parameters the number of lagged forecast errors in the prediction equation. (3) d the number of differences. This amounts to a total of (p,d,q) parameters to be estimated. These parameters are always estimated on using the stationary time series which is stationary with respect to its variance and mean.

To identify the appropriate ARIMA model for a time series, we have to identifying the order(s) of differencing needing to stationary the series and remove the gross features of seasonality, perhaps in conjunction with a variance-stabilizing transformation such as logging or

deflating. An notation AR(p) refers to the autoregressive model of order p. An AR(p) model is defined as

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + \varepsilon_t \quad (13)$$

where  $\phi_1, \phi_2, \dots, \phi_p$  are the parameters of the model, c is a constant and  $\varepsilon_t$  is white noise. An MA model is a common approach for modeling univariate time series models.

$$x_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q} \quad (14)$$

where  $\mu$  is the mean of the series, the  $\theta_1, \dots, \theta_q$  are the parameters of the model and the  $\varepsilon_t, \varepsilon_{t-1}, \dots$  are white noise error terms. The value of q is called the order of the MA model. An ARMA consists of two parts, an autoregressive (AR) part and a moving average (MA) part. The model is usually then referred to as the ARMA(p,q) model where p is the order of the autoregressive part and q is the order of the moving average part.

$$X_t = c + \phi_1 X_{t-1} + \dots + \phi_p X_{t-p} + \phi_1 \varepsilon_{t-1} + \dots + \phi_q \varepsilon_{t-q} + \varepsilon_t \quad (15)$$

An ARIMA model is a generalization of an ARMA model. The model is generally referred to as an ARIMA(p,d,q) model where p, d, and q are non-negative integers that refer to the order of the autoregressive, integrated, and moving average parts of the model.

#### 4. Empirical analysis

Decisions make in advance of uncertain events. And forecasting is an important for all strategic and planning decisions in a supply chain. Especially, forecasts of product demand, materials are important inputs to scheduling, acquiring resources, and determining resource requirement.

We use the data of the medium and small-sized business in real world and check the effectiveness of a suggested model. This company accepts an order from the upstream company. Because there is a change in this order, this company must have many stocks to avoid lack. As a result, this company must risk a burden for a cost to maintain a stock. Therefore, it is important that the company predicts quantity of order from the upstream company. If we predict the volume precisely, this company reduces the inventory cost. The reduction of the inventory cost in the model that we suggested had a big influence on the improvement of corporate earnings.

##### 4.1 Data

It assumes that the forces which influenced past value will continue to exist in the future. In this section, we follow the forecast methodology to calculate the numerical value of demand and price. We follow the forecast methodology to calculate the numerical value of demand and price. For the

numerical experiment, we use 122 weeks of data (from November, 2004 to December, 2008) for estimation by using Geometric Brownian motion, ARIMA model and mean Reversion.

#### 4.2 Empirical results of Geometric Brownian motion

In this research, we apply to both forecasting models to the forecast the demand of one die casting company and the price of copper. We use the data from November 2004 to December 2008. The actual demand shows the seasonality in the data series. For the actual price of copper, the data set shows high volatility and covers the period of the financial crisis.

##### (1)Results of demand

For the illustrative purpose, we simulate demand 1000 rounds for each period by using MATLAB. Figure 1 is a result from real option method. Real option method provides the forecasting value along with actual value as well. This method can capture demand changes quite accurately.

All the data were acquired on statistical software for economic analysis, called EViews, which is developed by Quantitative Micro Software (QMS). We analyze demand and price data which are separated to be 122 numbers of data. We follow the parameter setting from November 2004 to December 2008.

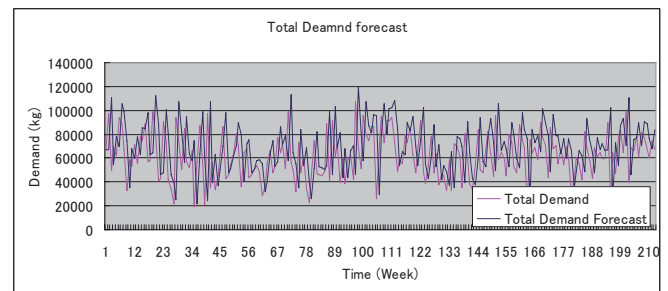


Figure 1 Demand forecast

##### (2)Results of price

We try to find the simulation result of mean reversion in price with 1000 times estimation.

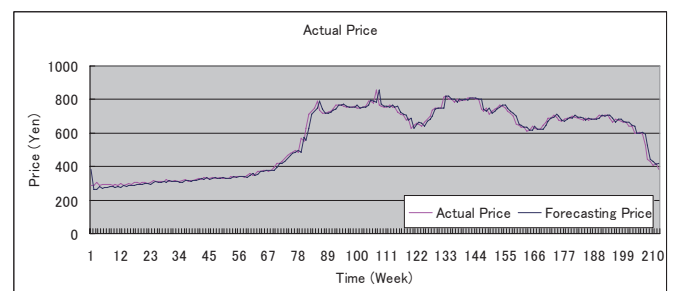


Figure 2 Price forecast

### 4.3 Empirical results of ARIMA model

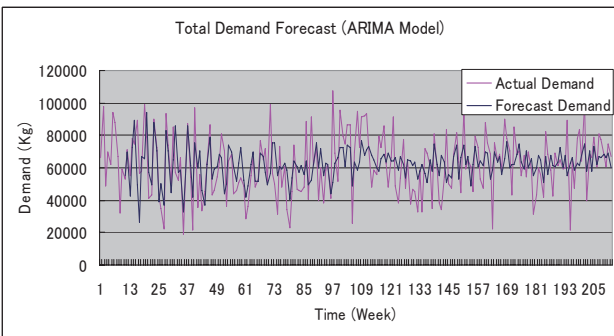
#### 4.3.1 Numerical experiment of Demand following real option method

We demonstrate a result of AIC in Table 2 with the different value of d in ARIMA (p,d,q) model. The best ARIMA (p,d,q) parameters for ARIMA model for demand is computed as ARIMA (9,1,9) that AIC is 22.4. We assume to use ARIMA (9,1,9) which offers the best Akaike information criterion for our simulation.

We demonstrate our forecasting result by using ARIMA (9,1,9) in Figure 3 as below. In the beginning ARMA model can produce demand forecast quite well but in the long run the forecast accuracy is getting lower.

**Table 2 ARIMA parameter for total demand**

	1.1.X	2.1.X	3.1.X	4.1.X	5.1.X	6.1.X	7.1.X	8.1.X	9.1.X	10.1.X
X.1.1	22.53	22.54	22.55	22.44	22.52	22.52	22.55	22.54	22.55	22.55
X.1.2	22.52	22.46	22.55	22.45	22.52	22.49	22.56	22.55	22.51	22.54
X.1.3	22.53	22.53	22.56	22.54	22.53	22.49	22.48	22.49	22.51	22.52
X.1.4	22.55	22.51	22.42	22.44	22.43	22.47	22.46	22.51	22.51	22.51
X.1.5	22.54	22.52	22.43	22.45	22.42	22.41	22.4	22.41	22.44	22.41
X.1.6	22.51	22.52	22.44	22.41	22.43	22.4	22.38	22.43	22.52	22.41
X.1.7	22.52	22.5	22.46	22.41	22.41	22.44	22.44	22.46	22.53	22.46
X.1.8	22.53	22.52	22.46	22.41	22.46	22.42	22.48	22.46	22.42	22.43
X.1.9	22.54	22.52	22.45	22.42	22.43	22.43	22.43	22.45	22.4	22.4
X.1.10	22.52	22.5	22.52	22.43	22.49	22.5	22.48	22.51	22.43	22.41



**Figure 3 Price forecast (ARIMA (9,1,9))**

The first step to simulate the forecasting price by using ARIMA model is to find the best AIC as shown in Table 3. We use ARIMA (6,1,3) model which provides the best AIC, 8.72. The 1000 round simulation result is shown in Figure 4. We can use ARIMA model to forecast the low volatility price accurately through all period.

#### 4.3.2 Comparison of demand forecasting

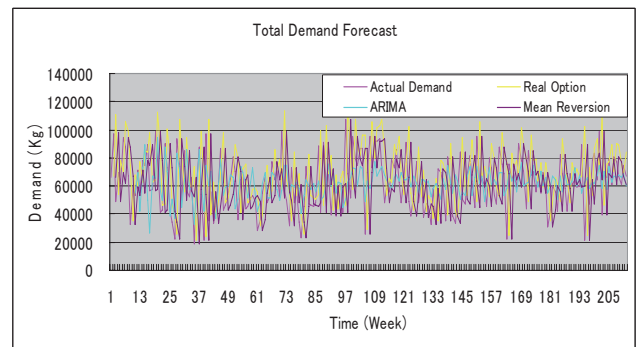
When we compare the result of forecasting between real option method and ARIMA model, we see that both method give quite similar degree of accuracy.

**Table 3 ARIMA parameter for total price**

	11X	21X	31X	41X	51X	61X	71X	81X	91X	101X
X11	8.902	8.898	8.898	8.911	8.926	8.939	8.9503	8.9581	8.970	8.980
X12	8.892	8.901	8.907	8.921	8.934	8.904	8.9572	8.9713	8.943	8.973
X13	8.897	8.904	8.916	8.93	8.907	8.72	8.9324	8.9479	8.949	8.955
X14	8.9	8.913	8.922	8.936	8.912	8.917	8.8992	8.9239	8.906	8.937
X15	8.908	8.919	8.931	8.91	8.913	8.747	8.7236	8.925	8.927	8.943
X16	8.918	8.928	8.939	8.904	8.919	8.94	8.9138	8.9272	8.948	8.941
X17	8.928	8.938	8.912	8.891	8.938	8.933	8.8775	8.9365	8.96	8.95
X18	8.938	8.903	8.92	8.92	8.929	8.927	8.932	8.9493	8.9462	8.91656
X19	8.927	8.918	8.924	8.919	8.919	8.911	8.9324	8.9511	8.9506	8.93665
X110	8.926	8.938	8.93	8.906	8.922	8.935	8.9436	8.8277	8.9519	8.94281

However real option approaches does a slightly better performance than ARIMA model. Demand data does not show high degree of fluctuation. This is an important condition that is contributed to the level of accuracy of in each method.

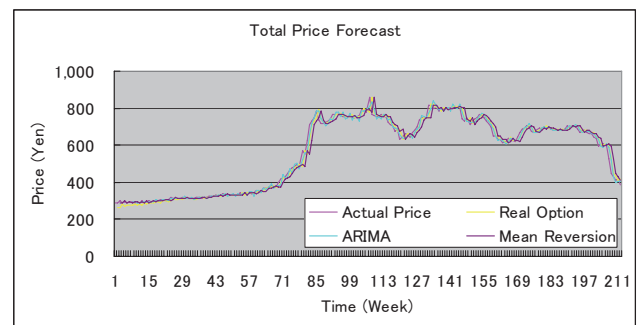
To verify the precision of forecasting model, we compare the actual and forecast result again. From Figure 4 and Figure 5, the real option method presents the best forecast accuracy.



**Figure 4 Demand forecasting result**

#### 4.3.3 Comparison of price forecasting

The price forecasting results from real option method and ARIMA model are very similar as shown in Figure 5. We prefer the real option model than ARIMA model because real option model is comparatively give a better level of accuracy to the actual price.



**Figure 5 Actual and forecast price**



This can be explained by the name of the change in price data. The price data has a higher level of volatility comparing the level of volatility of demand. This explains why Real options approach outperforms ARIMA. In short, we can say that in case of high volatility data Real options approach is preferred over ARIMA.

We show the comparison of the absolute deviation of demand and price in table 4 and table 5.

**Table 4 Absolute deviation of demand**

Real Option% AD	ARIMA Model%AD
0.182	0.168

**Table 5 Absolute deviation of price**

Real Option% AD	ARIMA Model%AD
0.06	0.6

## 5. Conclusion

In this study we compare the forecasting methods that are used to forecast demand and price. We use real option approach forecasting method that is based on Geometric Brownian motion for the prediction. This model is good in capturing the trend and the volatility of the data, which are the common characteristic of many data type especially in finance and supply chain management. The model is used to predict in a short term period. We highlight and compare the result with ARIMA method that is one of the famous models used to forecast time series. The result shows that real options approach perform better in both sets of data that we used to test. Especially when the data has a high volatility, real options shows less deviation than ARIMA model. Besides that the time interval and availability of data set also have a crucial impact on the accuracy of the forecast. Deseasonality of data and data segmentation would probably improve the performance of the forecast and still remain an interest of our future research.

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